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Effect of growth and development on cephalometric shapes in orthodontic patients: a Fourier analysis

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SUMMARY The age- and gender-related shape variations of the craniofacial skeleton in skeletal Class I children were quantified using a Fourier analysis on the pre-treatment lateral head films of 122 orthodontic patients (age range 7–15 years), who were subdivided into six groups for sex and age (2-year intervals). Seven landmarks representative of the maxillo-mandibular sagittal and vertical relationship were identified and digitized. The contiguous landmarks were connected by segments, the form was normalized with respect to its orientation and size, and a Fourier analysis of the contour was performed. Mean values of the cosine and sine coefficients of the first six harmonics in the sex and age classes were computed. The size-standardized outlines of the oldest boys were narrower and longer than the outlines of the youngest boys (differences at gonion, menton, sella and nasion). Shape differences between mean plots in girls were negligible. In the youngest patients, girls had a larger size-independent shape in the mandibular region; their shape was narrower (anterior–posterior direction) and longer (vertical direction) than male shape. In the oldest patients, boys had a larger size-independent shape at gonion, and a narrower shape at articulare and pogonion than girls. Size increased from the youngest to the oldest boys; size differences were not conspicuous in girls. Within an age class, male size was always larger than female. Fourier analysis allowed a global evaluation of the cephalometric forms, with separate quantifications of the age- and gender-related differences in size and shape.

Introduction

The evaluation of orthodontic patients is usually made from lateral cephalometric radiographs (Ricketts, 1981; Merow and Broadbent, 1990). Linear and angular measurements are performed starting from sets of standardized landmarks individualized on the films and compared to normative data (Broadbent *et al.*, 1975). Unfortunately, the investigated morphological changes affect the form of craniofacial structures in their entirety, i.e. both their size and shape components. Shape refers to the structure independently of its orientation, relation to reference planes, and dimension (or size) (Kuhl and Giardina, 1982; Lestrel, 1989). The conventional metric measurements provide quantitative information about size only, eluding the quantitative definition of shapes and of shape modifications, and reducing the chance of a diagnosis truly related to the global morphology of the face (Dung and Smith, 1988; Lestrel, 1989; Halazonetis *et al.*, 1991; Lestrel and Kerr, 1993; Lowe *et al.*, 1994). Not only is it difficult to separate the size and shape components of craniofacial structures, but also different combinations of cephalometric landmarks could produce the same linear or angular values (Yang and Suhr, 1993). This could be the effect of varying combinations of size and shape changes (Fine, 1994).

Indeed, several cephalometric analyses have been proposed for the quantitative analysis of the radiographic projection of craniofacial structures (Merow and Broadbent, 1990). Apart from the intrinsic limitations of the two-dimensional projections of three-dimensional structures, linear measurements sufficiently evaluate the dimensions of soft and hard tissue components. Since most analyses are mainly concerned with craniofacial shape and with the reciprocal arrangement of structures, a considerable number of angles and linear ratios are currently used.

Alternative analyses evaluate the intrinsic proportional form characteristics of a single patient avoiding any external reference. Enlow's counterpart analysis (Enlow *et al.*, 1969, 1971; Merow and Broadbent, 1990; Martone *et al.*, 1992) and Moorrees' mesh diagram analysis (Moorrees and Lebret, 1962; Moorrees *et al.*, 1976; Lebret, 1985) provide both qualitative and quantitative information on the relative proportions between the different parts of face and skull. These two methods are independent of the patient's size, but nevertheless they still do not quantify shape in an objective and absolute way, i.e. it is not possible to quantify the shape difference between two cases.

Cephalometric measurements are usually treated as single isolated variables, while the characterization of biological objects should always involve a multivariate analysis of the aspects and differences (Dung and Smith, 1988; Ferrario *et al.*, 1991). Multivariate analysis can better explain the effects of growth and development, the gender-related interactions, or the results of a specific treatment, on the hard and soft tissue structures of the craniofacial complex.

More sophisticated mathematical methods allow a multivariate analysis of craniofacial forms, where a correct quantitative analysis of shape and of its changes, together with a more ready account of size, are provided. In the analysis of craniofacial forms, both homologous point representations (Moss *et al.*, 1985; Lestrel, 1989; Bookstein, 1991; Fine and Lavelle, 1992) and boundary representations (Lu, 1965; Lestrel, 1989; Ferrario *et al.*, 1991, 1992, 1995a,b, 1996; Halazonetis *et al.*, 1991; Lestrel and Kerr, 1993; Lowe *et al.*, 1994) have been used. Cephalometric landmarks are used directly in homologous point methods, while in boundary methods they are used to reconstruct an outline mathematically.

Boundary representations deal with the outlines of the objects: a curve-fitting procedure (usually a harmonic analysis such as the Fourier series) is used to compute a mathematical function which will describe the object's outline, and which will be used to compare different objects, or the same object through time. Complex forms are thus decomposed into a series of cosine and sine functions of increasing frequency (Johnson *et al.*, 1992). Both classic Fourier series and elliptic Fourier analysis have already been used successfully for quantitative studies of biological forms in several fields: neurology, dentistry, osteology, haematology (Lu, 1965; Lestrel *et al.*, 1977; Johnson *et al.*, 1985, 1992; Diaz *et al.*, 1989; Lestrel, 1989; Casanova *et al.*, 1990; Ferrario *et al.*, 1991, 1992, 1995a,b, 1996; Halazonetis *et al.*, 1991; Lestrel and Kerr, 1993; Lowe *et al.*, 1994). These methods analyse the global shape characteristics of an object, and control for size differences, differing spatial orientations, and the dependency on reference planes. In particular, size could be a confounding factor in the analysis of shape changes, because its modifications are often of greater magnitude than the corresponding ones of shape (Lestrel *et al.*, 1977; Bookstein, 1991; Lestrel and Kerr, 1993; Ursi *et al.*, 1993).

In this investigation, the aim was to analyse and describe the intrinsic (i.e. size- and orientation-independent) morphological characteristics of the radiographic image (lateral projection) of the craniofacial skeleton. The Fourier analysis (Lu, 1965; Lestrel *et al.*, 1977) was used to assess the effect of age and gender on craniofacial shape in skeletal Class I children. Size variations were also quantified separately.

Materials and methods

Sample

The pre-treatment lateral head films of all patients aged 7–15 years attending a private practice for orthodontic treatment in a 2-year period were evaluated. The 337 patients were all northern Italian (white Caucasians), and had dental malpositions correctable by orthodontic treatment only, as judged by an expert orthodontist. No surgical patients were evaluated. The ANB (subspinal–nasion–supramentale) angle was computed in all patients and corrected for both maxillary position (sella–nasion–subspinal angle, SNA) and rotation of the jaw (sella–

nasion–mandibular plane angle, SN-GoMe), as proposed by Miralles *et al.* (1991) and detailed by Ferrario *et al.* (1994b). The corrected ANB angle was computed as: original ANB angle + 0.5 × (81.5° – SNA angle) + 0.25 × (32° – SN-GoMe angle). Only the skeletal Class I patients (corrected ANB angle between 0 and 4 degrees) were further analysed. One hundred and twenty-two patients (50 boys and 72 girls, mean age 10.2 years) met the selection criteria of age and skeletal Class I. The composition of the sample is reported in Table 1.

Radiographic technique and cephalometric measurements

Lateral cephalometric radiographs were taken in the standing posture, using the Orthoceph 10E (Siemens AG, Germany), 18 × 24 cm films, and a final enlargement of 10 per cent. The radiographic technique has been described in detail previously (Ferrario *et al.*, 1994a).

On all films, seven landmarks (nasion, sella, articulare, gonion, menton, pogonion, anterior nasal spine) were selected. The choice of these seven landmarks was arbitrary, being selected in order to describe an orthodontically significant form that would consider the maxillo-mandibular relationships in both the anterior–posterior and vertical directions (Siriwat and Jarabak, 1985; Nanda, 1988; Ferrario *et al.*, 1991). A single operator traced and digitized the points by means of a semiautomatic image analyser interfaced to an AT computer.

Fourier analysis

All the subsequent analysis was performed by a computer programmed by one of the authors. The method is as follows.

Centroid calculation. The contiguous landmarks were connected by segments (Figure 1),

Table 1 Number of subjects in each class, and mean and standard deviation of the cosine (*a*) and sine (*b*) coefficients of the first six harmonics in the sex and age classes.

H	*a*	SD	*b*	SD	*a*	SD	*b*	SD
7–9 years (boys, n = 16; girls, n = 27)								
0	42.96362	0.1848451			43.03698	0.2105969		
1	–0.51623	0.0909514	0.17694	0.0611880	–0.62119	0.1614015	0.25989	0.1214466
2	–3.09349	0.7848394	–0.89665	0.8698154	–4.11639	1.1468106	–0.72083	1.1247000
3	–3.85936	0.3104428	1.49011	0.6434743	–3.88290	0.4319135	1.79974	0.7566622
4	–2.37037	0.5116495	0.70987	0.4685192	–1.98450	0.6717123	0.87349	0.3655589
5	1.89024	0.2865753	–0.53873	0.6076162	2.10530	0.3155594	–0.83229	0.6981036
6	1.49343	0.2682355	–0.72249	0.4467222	1.45779	0.3379635	–0.92160	0.4332572
10–12 years (boys, n = 19; girls, n = 27)								
0	42.89733	0.1775990			42.97154	0.2244278		
1	–0.65275	0.1594878	0.22244	0.1393333	–0.64536	0.1184591	0.22181	0.1088306
2	–4.16709	1.3114081	–0.24021#	0.9184365	–4.06902	0.7812123	–0.38129	0.9559420
3	–3.86891	0.4722110	1.87251	0.8804563	–3.89691	0.3932321	1.82255	0.8814070
4	–1.92845	0.8412856	0.77252	0.4342698	–1.94180	0.7056624	0.84884	0.3560031
5	1.81746	0.3262499	–0.88099	0.8051055	1.94399	0.3461181	–0.88285	0.7211406
6	1.48631	0.4118313	–1.07470	0.3992106	1.43542	0.4198838	–1.08257	0.4307232
13–15 years (boys, n = 15; girls, n = 18)								
0	42.91026	0.2347622			42.95648	0.2613182		
1	–0.58588	0.1728323	0.20843	0.1182326	–0.64816	0.1652246	0.24905	0.1429453
2	–4.54409	1.7571265	–0.40680#	1.0941394	–4.29014	1.1323774	–0.65580#	1.3336115
3	–3.37392	0.1976314	1.11629	1.0355932	–3.85615	0.5856600	1.42774	1.1683518
4	–1.86212	1.0928584	0.79256	0.5927880	–1.99262	0.7760024	0.79959	0.4120997
5	2.00402	0.4414632	–0.82677	0.8724017	1.95043	0.3556577	–0.61255	0.8475529
6	1.51334	0.1860182	–0.75614	0.6415149	1.55038	0.5035909	–0.89104	0.5362100

All the coefficients are significant within groups ($P \leq 0.05$) unless noted (#).

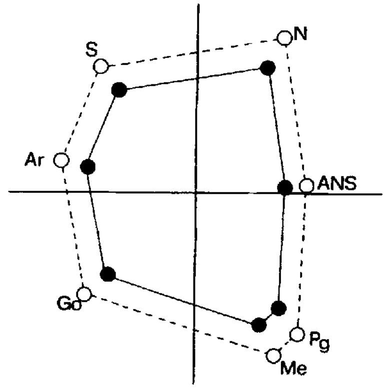

Figure 1 Cephalometric landmarks used in the Fourier analysis (N: nasion, S: sella, Ar: articulare, Go: gonion, Me: menton, Pg: pogonion, ANS: anterior nasal spine). The landmarks are the endpoints of the analysed form. Open circles and interrupted line represent digitized landmarks and form with its original size. Closed circles and continuous line: size-standardized form used for shape analysis only. The centre of gravity and the coordinate axes are also indicated.

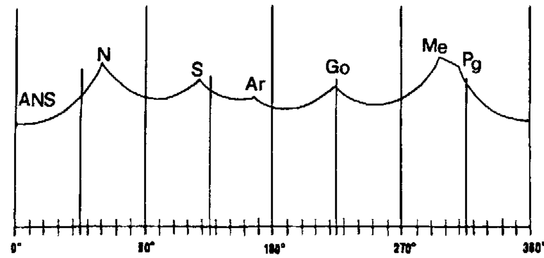

Figure 2 Vector modulus (Y-axis) as a function of the vector angle (X-axis) in the plot shown in Figure 1.

and an area-weighted centroid (or centre of gravity, CG) of the form thus delimited was calculated using the x, y coordinates of the cephalometric landmarks, and the area delimited between pairs of contiguous landmarks. The centroid of the form was set as the origin of cartesian axes.

Normalization. The form was normalized with respect to its orientation relative to the coordinate axes, and to its size. The origin of the axes was set in the centroid of the contour, and the form was orientated with the sella–nasion line rotated at +10 degrees relative to the horizontal axis, this line thus intersecting the Y-axis. This value was arbitrarily chosen for representation purposes only, being the average value when the head is orientated in the standard cephalometric position (hard tissue Frankfort plane parallel to the ground) (Ferrario *et al.*, 1994a). The form was then size-normalized by setting its area equal to a constant of 5000 mm^2. This arbitrary value corresponds to the average area of the enclosed profile on a standard teleradiograph (enlargement 10 per cent, Ferrario *et al.*, 1994a), and allows a convenient graphical representation. Size normalization was performed mathematically with expansion or contraction of the area that did not modify the shape (continuous line in Figure 1).

Curve fitting and harmonic analysis. From the CG (origin of axes), a vector was rotated counterclockwise for 360 degrees with a 1 degree step to intersect the contour of the form, and 360 segments were produced. The magnitude of each vector (modulus, or distance from the CG to one of the 360 points of the contour), together with the relevant vector angle relative to the horizontal, was used to generate a curve in a cartesian coordinate system: y, vector modulus; x, angle (from 1 to 360 degrees) (Lestrel *et al.*, 1977; Halazonetis *et al.*, 1991). Figure 2 shows the vector modulus as a function of the vector angle in the same tracing of Figure 1. Using the least squares method, a Fourier analysis of the curve was thus performed, with period τ = 360 (Lu, 1965).

The Fourier series expansion is defined as:

$$y = \frac{a_0}{2} + \sum_{m=1}^{6}\left[a_m \cdot \cos(\theta \cdot m \cdot x) + b_m \cdot \sin(\theta \cdot m \cdot x)\right]$$

where m is the harmonic, a_m is the cosine coefficient of m harmonic, b_m is the sine coefficient of m harmonic, $\theta = 2\pi/\tau$, with τ = normalized outline length.

Previous investigations (Ferrario *et al.*, 1995a, 1996) have already shown that the first six sine and cosine components in the Fourier series explain the analysed cephalometric shapes almost completely. Series were thus truncated at the 6th harmonic.

The difference between pairs of cephalometric plots was computed separately for the size and shape components: size difference was appreciated from the original (before size standardization) area ratio, shape difference was computed from the Fourier reconstruction of the outline (Ferrario *et al.*, 1995b, 1996).

Goodness of fit. The goodness of fit for each individual curve, i.e the agreement between the observed/digitized values and the values estimated according to the Fourier series, was calculated as proposed by Lu (1965) from the variances of the estimated and observed data. According to Lu (1965), this coefficient of agreement should be at least 0.6, thus indicating a formal correlation of approximately 0.9 between the observed and estimated data.

Graphic subroutine. A graphic subroutine allowed the qualitative control of the whole procedure. The subroutine plotted the cephalometric landmarks, the digitized outline and its harmonic reconstruction performed by summating the single functions of the series (Figures 1 and 2).

Statistical comparisons. Descriptive statistics (mean and standard deviation) for each coefficient were calculated within sex and age class from the coefficients of each cephalometric tracing. The significance of the coefficients was evaluated with a factorial analysis of variance (Lu, 1965).

A further two-way factorial analysis of variance compared the effects of (1) gender, (2) age and (3) the interaction gender × age for all the significant coefficients. Significance was set at the 5 per cent level ($P \leq 0.05$).

Error of method

The only significant source of error of this investigation (provided the films were of good quality) was in landmark location and digitization. Fifteen randomly selected radiographs were retraced and redigitized by the same investigator. Differences between pairs of repeated tracings and digitizations were computed separately for the size (1) and shape (2) components: a Fourier analysis of the forms individualized by the seven selected landmarks was performed, and pairs of forms compared (1) from the area ratio (unit: mm^2/mm^2) of the original cephalometric tracings, and (2) from the size-, orientation- and rotation-normalized Fourier reconstructed outlines (differential area between two normalized outlines expressed as a percentage of the sum of the relevant areas, unit: %). The repeated digitizations (of the same traces) gave shape differences ranging between 0.18 and 0.54 per cent, and size differences between 0.99 and 1.01. The repeated tracings (of the same radiographs) gave differences between 0.58 and 0.88 per cent (shape), and between 0.98 and 1.02 (size). Other negligible errors could derive from the mathematical procedure, in particular from the approximation algorithms.

Results

In all cephalometric tracings, the coefficient of agreement (Lu, 1965) was higher than 0.91 when the series were truncated at the 6th harmonic. Superimposition between the original plot and its mathematical reconstruction was also tested qualitatively with the graphic subroutine.

Table 1 reports the mean and standard deviation of the cosine (a) and sine (b) coefficients of the first six harmonics computed in the six age and sex classes. All the coefficients were significant within groups, except the sine coefficient of the second harmonic b_2 in boys 10–12 years, and in both boys and girls 13–15 years. Indeed, variability within age and sex classes was limited, especially for cosine coefficients: the coefficients of variation (the percentage ratio of the standard deviation to the mean) ranged between 0.5 and 60 per cent. In boys, variability increased as a function of age, while no similar differences were observed in girls. The largest variability was observed for coefficients a_4 regardless of gender or age. Conversely, sine coefficients were less homogeneous within class, b_2 and b_5 being the most variable. In girls, the oldest age class showed the highest coefficients of variation. Low statistical variations reflect almost consistent skeletal patterns within a group, while high variations reveal a relatively more scattered morphology.

Table 2 reports the results of the two-way analysis of variance calculated for all the within-group significant coefficients. The cosine

Table 2 Two-way factorial analysis of variance between the six age and sex classes.

Harmonics	1st		2nd		3rd		4th		5th		6th	
	F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
Coefficient *a*												
Sex	2.83	ns	2.26	ns	1.36	ns	0.64	ns	3.96	*	0.07	ns
Age	2.42	ns	2.79	ns	1.31	ns	0.84	ns	1.76	ns	0.39	ns
Sex × age	1.69	ns	3.39	*	2.45	ns	1.13	ns	0.93	ns	0.12	ns
Coefficient *b*												
Sex	3.01	ns	—		0.44	ns	1.36	ns	0.14	ns	0.94	ns
Age	0.14	ns	—		2.60	ns	0.01	ns	0.81	ns	3.41*	
Sex × age	1.22	ns	—		0.72	ns	0.29	ns	0.91	ns	0.68	ns

The *b* coefficient of the second harmonic was not compared because three of six coefficients were not significant within groups (Table 1). *F*: variance ratio; *P*: probability value; ns: not significant ($P > 0.05$); $*P \leq 0.05$. Degrees of freedom: sex 1,116; age 2,116; interaction sex × age 2,116.

coefficient of the 0-harmonic a_0 was not compared because the plots were size normalized, thus making all a_0 similar apart from digitization and approximation errors. Significant effects were found for gender, cosine coefficient a_5, age, sine coefficient b_6, and gender × age, cosine coefficient a_2. It seems that gender and age variations involve different aspects of the mathematically reconstructed craniofacial hard tissue morphology.

To separate the contributions of gender and age to the observed differences in the Fourier coefficients, size and shape differences were computed between the mean tracings of the six classes (Table 3). Size differences were computed from the area ratio (unit: mm^2/mm^2) of the original cephalometric tracings, while shape differences were calculated between the size-, orientation- and rotation-normalized Fourier reconstructed outlines. They corresponded to the differential area between two normalized outlines expressed as a percentage of the sum of the relevant areas (unit: %). In boys, size increased from the youngest to the oldest class; however, size in the 7- to 9-year-old girls was similar to that in 10- to 12-year-old girls (area ratio 0.99). Within age class, male size was always larger than female: the difference was particularly significant in the 10- to 12-year-old group.

Shape differences during the analysed age interval were evident in the boys, but negligible in girls. Gender influenced shape in the youngest (boy:girl difference 1.69) and oldest (boy:girl difference 1.04) age classes, while no shape differences were noted in the 10- to 12-year-old children.

A suggestive qualitative control of the shape differences was performed superimposing the size-standardized mean plots of children of (1) the same sex in different ages and (2) the same age in different sexes. In this way, only the shape differences were observed, independently from size.

The outlines of the oldest boys (10–12 and 13–15 years) were narrower and longer than the outlines of the youngest boys (7–9 years): the main differences involved the landmarks gonion and menton (mandibular plane), and sella and nasion (cranial base). To appreciate the main differences and similarities in shape, the size-standardized mean plots of the youngest and oldest children within sex were plotted (Figure 3), together with the magnitude and direction of the differences between the Fourier coefficients computed in the two ages. In the male plots (Figure 3a), the shape of the oldest children was larger at the mandibular base (landmarks menton and gonion, grey area in the lower part of the plot), and in the anterior cranial base (nasion and sella, grey in the upper part of the plot) than the shape of the youngest children.

Table 3 Size and shape differences between the mean cephalometric tracings in the six age (7–9, 10–12 and 13–15 years old) and sex (M: boys, F: girls) classes.

Group difference	M 10–12		M 13–15		F 7–9		F 10–12		F 13–15	
	Shape	Size	Shape	Size	Shape	Size	Shape	Size	Shape	Size
M 7–9	2.11	1.12	2.53	1.41	1.69	0.93	–	–	–	–
M 10–12	–	–	1.48	1.25	–	–	0.37	0.82	–	–
M 13–15	–	–	–	–	–	–	–	–	1.04	0.87
F 7–9	–	–	–	–	–	–	0.58	0.99	0.75	1.32
F 10–12	–	–	–	–	–	–	–	–	0.86	1.32

Size is the column:row area ratio (unit: mm^2/mm^2), shape is the difference between the size-, orientation- and rotation-normalized Fourier reconstructed outlines (unit: %). Comparisons were performed within sex between age class, and within age class between sex.

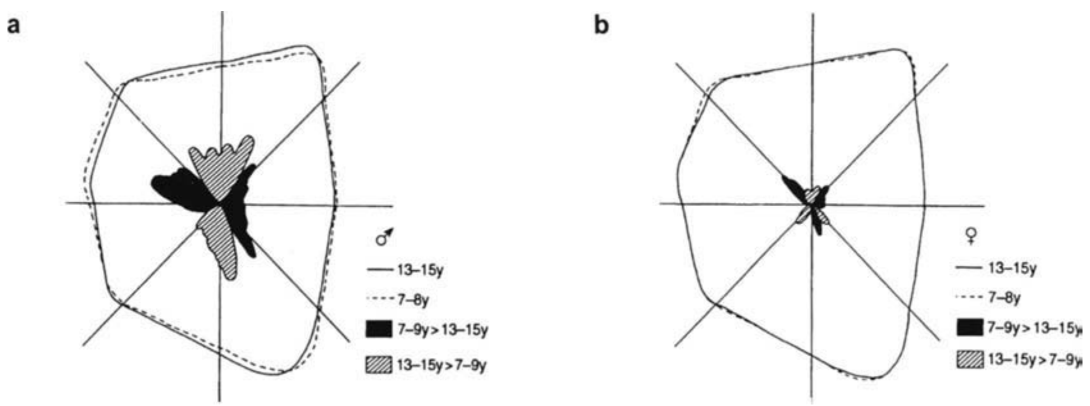

Figure 3 Size-normalized Fourier reconstruction of the mean tracing in the 7- to 9-year-old group (interrupted line) and in the 13- to 15-year-old group (continuous line). Series were truncated at the 6th harmonic. The magnitude of the differences between Fourier coefficients and their directions are indicated with a ×10 magnification (grey: 13–15 year coefficients larger than 7–9 year coefficients; black: 13–15 year coefficients smaller than 7–9 year coefficients). (a) Boys; (b) girls.

In the female group (Figure 3b), the differences in shape between mean plots were negligible: small differences were observed at the sella, gonion and menton landmarks. Indeed, the size-standardized plots were practically superimposed.

In the 7- to 9-year-old age class, the main differences were located at the pogonion, menton, sella and nasion landmarks. When standardized for size, girls had a larger shape in the mandibular region (chin and mandibular base); overall, their cephalometric shape was narrower (anterior–posterior direction) and longer (vertical direction) than male shape of the same age class (Figure 4a). In the 13- to 15-year-old age class, the pogonion, menton, articulare and gonion landmarks showed the largest gender variations. When standardized for size, boys had a larger shape at the gonion, and a narrower shape at the articulare and pogonion than girls of the same age class (Figure 4b). Conversely, the size-standardized plots of the 10- to 12-year-old children showed no gender differences in shape.

Discussion

Fourier analysis has already been used to

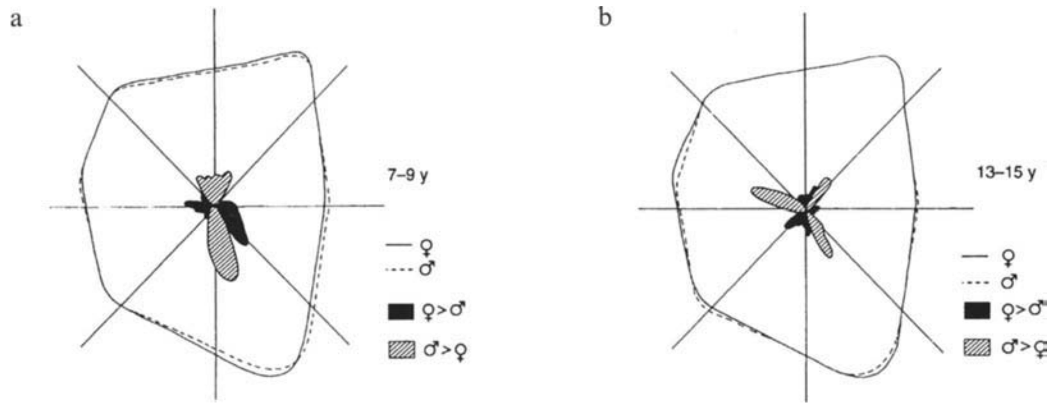

Figure 4 Size-normalized Fourier reconstruction of the mean tracing in the 7- to 9-year-old group (a) and in the 13- to 15-year-old group (b). Boys: interrupted line; girls: continuous line. Series were truncated at the 6th harmonic. The magnitude of the differences between Fourier coefficients and their directions are indicated with a ×10 magnification (grey: male coefficients larger than female coefficients; black: male coefficients smaller than female coefficients).

quantify the differences in shape in cephalometric tracings. Lestrel and Kerr (1993) applied the elliptic Fourier series to quantify the effect of functional therapy on the maxillary and mandibular shape. Fourier series were used to describe the shape and the shape changes of the outline of these bones on a lateral cephalometric radiograph. Only a limited number (14) of patients with a Class III malocclusion were investigated. Lowe *et al.* (1994) analysed a large number of patients older than 14 years of age. Elliptic Fourier series quantified the shape of the maxilla, mandible and cranial base. The consistency between the conventional cephalometric classification of anterior–posterior and vertical relationships, and clustering according to Fourier descriptors was evaluated. Indeed, the two criteria were only marginally in agreement, with an overall error rate of 33 per cent. A similar comparison between Fourier elliptic classification and divergence classification was performed by Ferrario *et al.* (1991).

Two limitations of these studies can be pointed out. Firstly, in both the Lestrel and Kerr (1993) and Lowe *et al.* (1994) investigations the radiographic shadows of the bones were completely traced. This procedure seems of limited practical use: not only are the outlines of the bones not always well defined, but orthodontists are more used to analysing sets of landmarks. Secondly, in all the studies, the shapes were analysed using the elliptic Fourier series. Indeed, the elliptic Fourier series could replace the classic Fourier series in the analysis of closed forms (Kuhl and Giardina, 1982; Lestrel, 1989; Ferrario *et al.*, 1991; Lestrel and Kerr, 1993; Lowe *et al.*, 1994). Unfortunately, in the elliptic analysis, the outline of the form has to be codified using a rather elaborate and complex algorithm. Moreover, each harmonic is an ellipse (Kuhl and Giardina, 1982; Lestrel and Kerr, 1993), and their geometrical effect on the contour of complex shapes could be difficult to interpret.

The method proposed in this investigation seems to be simpler and easier to understand, and it can be applied to both open and closed forms. Moreover, whenever groups of forms have to be compared, the analysis of variance for the comparison of Fourier coefficients (which satisfy the property of orthogonality, and can thus be further analysed with conventional statistics) has already been exemplified in detail by Lu (1965) and Lestrel *et al.* (1977) for the classic Fourier series.

A further advantage of the classic series is the geometric meaning of the coefficients of the harmonics: the numerical differences in the Fourier coefficients can be related to actual differences in the observed morphology, as illustrated by the relevant plots (Lu, 1965; Lestrel *et al.*, 1977; Ferrario *et al.*, 1996).

The classic Fourier series were applied to the study of craniofacial hard and soft tissue structures from cephalometric radiographs, standardized photographs and infrared three-dimensional reconstructions by Lu (1965), Halazonetis *et al.* (1991) and Ferrario *et al.* (1992, 1995a,b, 1996). The effects of growth and gender on the shape of single bones, and on the facial frontal and sagittal profiles were investigated. The analysis of the harmonics most relevant in explaining shape permitted localization of the main differences, and allowed them to be related to the direction of growth.

Classic Fourier series have some intrinsic limitations however. First, the method is suitable for every complex open or closed form provided that the intersection between the contour of the form and the vector is unique, i.e. the outlines of the form should not be excessively concave relative to its centroid. This could be a limitation in the analysis of the radiographic projection of single bones (Lestrel, 1989; Lestrel and Kerr, 1993; Lowe *et al.*, 1994). Secondly, according to Lestrel (1989), the method is not really coordinate free, because a centroid is needed, and a starting point should be defined (Lestrel *et al.*, 1977; Johnson *et al.*, 1985). Conversely, the elliptic method proposed by Kuhl and Giardina (1982) allows an internal orientation, performed rotating the forms until the major axes of the first harmonic ellipses coincide. In the present method, the orientation was necessarily identical for all the forms, and defined starting from the intrinsic skeletal (sella–nasion line) and geometrical [the area-weighted centroid, as used by Lestrel *et al.* (1977) and Johnson *et al.* (1985)] characteristics of each form independently from size. In this way, a type of internal orientation was used, according to the homology between the collected landmarks across the cephalometric tracings (Lu, 1965; Lestrel *et al.*, 1977; Johnson *et al.*, 1985). Fourier analysis can be a true shape analysis, because normalized Fourier coefficients are position, size and orientation invariant (Shen *et al.*, 1994). Standardization for size, position and orientation is an essential step in growth studies. Growth direction is usually assessed by superimposing the cephalometric tracings, and an incorrect positioning could influence the estimate (Lundström and Woodside, 1980). Growth studies often deal with individuals with great size differences, but size could play a distorting role in the estimation of shape (Nanda, 1992). The craniofacial tracings analysed in this investigation differed in both size and shape: Fourier analysis corrected for the size discrepancy, and separated the size and shape contributions to the global morphology.

Whilst the main aim of this investigation was the analysis of shape modifications, size differences were also quantified. Size differences in boys (Table 3) were consistent with the well-known modifications during the analysed period of growth (Bishara *et al.*, 1984; Ursi *et al.*, 1993), while size difference in girls were in part atypical: the girls in the 10–12 year age class were smaller than the 7- to 9-year-old girls. Their size difference relative to the boys of the same age class was larger than the differences observed in the other age classes (22 per cent instead of 8 per cent, and 15 per cent). Indeed, gender differences in the timing and magnitude of the modifications of facial linear and angular dimensions during growth have already been reported in longitudinal studies (Lundström and Woodside, 1980; Bishara *et al.*, 1984; Ursi *et al.*, 1993). The mean growth curve for both the anterior (nasion–gnathion) and posterior (sella–gonion) facial heights in girls aged 7–14 has been reported to be rather flat (Van der Beek *et al.*, 1991). The 'negative' growth found in our investigation may be due to the wide individual variations in growth (Nanda, 1992), which could be reflected in a relatively small sample (27 girls in each of the two first age classes), or a sampling problem: this investigation used transversal data collection, which is more prone to secular trends than a longitudinal study (Ursi *et al.*, 1993).

Unfortunately, the biological price to be paid for radiographic investigations is currently considered by international medical standards to be too high to allow new longitudinal studies. Classical investigations (Broadbent *et al.*, 1975) will become difficult to perform in the future.

The analysis of variance found some difference between the size-standardized Fourier coefficients (Table 2), but when the global outlines were compared, the differences partly compensated each other. In the analysed age interval, the craniofacial shape (as seen in a lateral cephalometric projection) was significantly modified in the male group only (Figure 3a). Conversely, shape modifications in girls were negligible (Figure 3b). Gender differences in shape were found in the youngest children (Figure 4a), but they were less evident in the oldest age class (Figure 4b).

The comparison of the present study with previous cephalometric analyses is difficult, since no other investigation has analysed similar shapes independently from size (Ursi *et al.*, 1993). The only study that applied a true shape analysis to the effect of growth on the global craniofacial outline in lateral cephalometric radiographs is reported by Ferrario *et al.* (1996). Indeed, the study was limited to the Bolton standard of dentofacial developmental growth (Broadbent *et al.*, 1975). As to the shape differences observed in the present male group, they were similar to those in the Bolton standards, representing a male–female average. Conversely, girls did not modify their craniofacial shape accordingly. Other investigations have reported the modifications of linear and angular measurements as a function of growth. The modification of angular measurements could, in a first approximation, supply an indication of shape changes, given all the limitations already mentioned in the Introduction. For instance, in a mixed-longitudinal study on Dutch girls 7–14 years of age, the anterior and posterior facial heights gradually increased, but the mandibular plane angle did not modify significantly (Van der Beek *et al.*, 1991). In a longitudinal investigation performed on American (Caucasians with North European ancestry) girls, Nanda (1992) found that, while the linear measurements did not follow a constant growth rate in the various parts of the face and cranium, their reciprocal relationships were maintained from childhood to adulthood. Angular measurements such as the sella–nasion–anterior nasal spine angle were nearly invariable in the analysed age span (Nanda, 1992). While some gender differences in craniofacial shape were found in the youngest children of the present investigation, Ursi *et al.* (1993) found no gender differences in several cephalometric angles measured in a longitudinal study of North American whites from 6 to 18 years of age. In those children, gender differences in size (maxillary and mandibular lengths) were evident only after 14 years of age, while in our children size differences (area of the analysed cephalometric form) were larger between 10 and 12 years of age than in other age classes.

The present investigation was restricted to a group of patients with a selected anterior–posterior skeletal relationship in a limited age range. Further applications of Fourier series could analyse the effect of different skeletal or aesthetic classifications on the age and gender variations of cephalometric shapes, as well as quantifying the effect of orthodontic and surgical therapy on selected malocclusions and facial malformations (Lestrel and Kerr, 1993; Lowe *et al.*, 1994).

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